MOTION OF A DISPERSE SYSTEM IN A CHANNEL WITH PERMEABLE WALLS

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The effect of filtration of the liquid phase through the channel walls on the hydraulic resistance is considered in the case of flow in a round tube. Depending on the flow parameters there may be either an increase or a significant reduction (up to 50–60%) in the hydraulic resistance in comparison with flow in a tube with impermeable walls.

The dynamic characteristics of a flow of a disperse system depend significantly on the filtration flow of the liquid phase through the channel walls. On one hand, filtration of the dispersion medium leads to an additional effective pressure gradient in the flow (as occurs in the flow of a uniform liquid [2]). This is manifested in a reduction of the observed hydraulic resistance in comparison with a flow in a channel with impermeable walls. On the other hand, removal of the liquid phase from the system leads to an increase in the concentration of the system and its effective viscosity, with the result that the hydraulic resistance increases.

A quantitative treatment of these effects is of interest primarily in connection with applications to several chemical manufacturing processes. In addition, the problem of flow in a channel with permeable walls can be regarded as a model of the flow of clay and cement solutions in a well sunk in a porous stratum, a situation of interest to the oil recovery industry.

It is necessary, generally speaking, to consider separately the equations of motion of the dispersion medium and the disperse phase even in regions of steady flow. There is no consistent mechanical scheme at present for doing this. Hence, we assume here that the disperse medium can be regarded as a dispersoid consisting of an ordinary viscous liquid with effective viscosity μ, which depends on the volume concentration ρ of the solid phase. The following conditions must be satisfied if this assumption is to be valid:

1. The medium must be sufficiently finely dispersed and (or) uniformly dense, so that the Stokes velocity of the particles is negligibly small in comparison with the mean flow velocity. This is the case in a considerable number of hydraulic and even pneumatic transport systems, as well as for the clay and cement solutions usually used in practice.

2. Slipping at the walls is insignificant. This assumption is usually valid if the diameter of the particles is much smaller than the linear dimensions of the cross section of the channel.

3. The density of the filtration flow of the dispersion medium is much less than the mean density of the main flow in the section. Then changes in the logarithms of all the quantities (except the pressure p) at distance on the order of the linear dimensions of the cross section along the channel axis are small in comparison with unity and the radial velocities are small in comparison with the velocity along the axis. In correspondence with this assumption we also regard the disperse medium as uniform in the radial directions. Considering the equations of motion of a dispersoid we see that we can neglect radial motion of the phases only in the zero approximation for small derivatives of the velocity and other quantities (except p) in the direction of the flow.

In these approximations the axisymmetric steady flow is

$$-\frac{dp}{dz}+\frac{\mu(\rho)}{r_1}\frac{\partial}{\partial r_1}\left(r_1\frac{\partial v}{\partial r_1}\right)=0,\quad v|_B=0. \tag{1}$$

Here B denotes the solid boundaries of the flow. For a flow in a round tube we obtain from (1) a Poiseuille velocity distribution. The equations of conservation of mass of the phases in this flow in integral form are

$$\frac{d}{dz}\left[(1-\rho)Q\right]+2\pi Rq=\frac{d}{dz}(\rho Q)=0,$$
$$Q=2\pi\int_0^R r_1 v(r_1,z)\,dr_1. \tag{2}$$

We regard the quantity q(z) as linearly dependent on the pressure drop on the tube walls. We then have

$$q(z)=k(p-\varphi),\quad \varphi=\varphi(z),\quad k=k(z). \tag{3}$$

This expression is valid for $q > 0$, or, if the tube is immersed in the liquid phase of the given disperse system, for any q. From (2) and (3) we obtain the equations

$$C=\rho Q=\text{const},\quad \frac{dQ}{dz}+2\pi Rk(p-\varphi)=0. \tag{4}$$

From (4) and from (1) we have the relationships for

$$-\frac{\rho R^4}{\mu(\rho)}\frac{dp}{dz}=C,$$
$$p=p_0-\frac{8C}{\pi}\int_0^z\frac{\mu(\rho)}{\rho}\frac{dz}{R^4}. \tag{5}$$

Using (4) and (5) we obtain a second-order equation for $\rho(z)$,

$$\frac{d^2\rho}{dz^2}-\frac{2}{\rho}\left(\frac{d\rho}{dz}\right)^2+\frac{d\rho}{dz}\frac{d\ln k}{dz}+$$
$$+\frac{2\pi k\rho^2 R}{C}\left(\frac{8\mu(\rho)C}{\pi\rho R^4}-\frac{d\varphi}{dz}\right)=0. \tag{6}$$

For simplicity we consider below only the case of flow in which φ, R, and k are constant. Solving (6), we obtain the equation

$$\frac{d\rho}{dz} = \rho^2 \left[k \left(4\pi^2 kR \frac{(p_0 - \varphi)^2}{\rho_0^2 Q_0^2} - \right.\right. \quad (7)$$

$$\left.\left. - \frac{32}{R^3} \int_{\rho_0}^{\rho} \frac{\mu(\rho)\, d\rho}{\rho^3} \right) \right]^{1/2}.$$

In deriving (7) we used the initial conditions

$$\rho|_{z=0} = \rho_0,$$

$$\left.\frac{d\rho}{dz}\right|_{z=0} = -\frac{2\pi kR}{C} (p_0 - \varphi)\rho_0^2, \quad C = \rho_0 Q_0. \quad (8)$$

We note that instead of (8) we can use any other conditions, e. g., we can assign the pressure at the outlet of the tube, and so on.

For the effective viscosity of a dispersoid we can use the empirical formula $\mu = \mu_0(\rho_* - \rho)^{-n}$, where the parameter n lies between 2 and 4, according to the data of different authors. For definiteness we take n = 3. Then, after integration we obtain from (7) ($\rho_* = \max \rho$) that

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho^2 \left[1 - \alpha\left(F(\rho) - F(\rho_0)\right)\right]^{1/2}} = \xi,$$

$$F(\rho) = \int \frac{d\rho}{\rho^3 (\rho_* - \rho)^3} =$$

$$= -\frac{1}{\rho_*^5} \left[6 \ln \frac{\rho_* - \rho}{\rho} - \frac{4\rho}{\rho_* - \rho} - \right.$$

$$\left. - \frac{\rho^2}{2(\rho_* - \rho)^2} + \frac{(\rho_* - \rho)^2}{2\rho^2} + \frac{4(\rho_* - \rho)}{\rho} \right]. \quad (9)$$

Here we have introduced the dimensionless variable and parameter

$$\xi = 2\pi kR \frac{p_0 - \varphi}{\rho_0 Q_0} z, \quad \alpha = \frac{8}{\pi^2} \frac{\mu_0}{kR^5} \left(\frac{\rho_0 Q_0}{p_0 - \varphi} \right)^2. \quad (10)$$

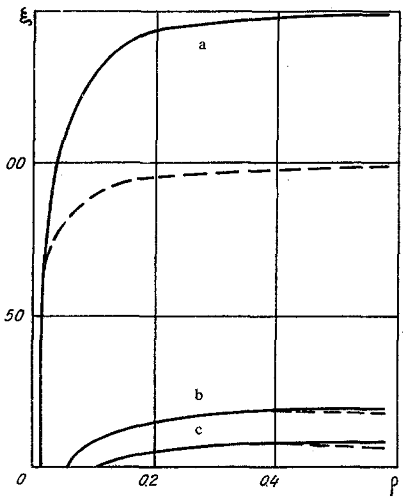

Fig. 1. Relationship $\xi = \xi(\rho)$ for $\alpha = 3 \cdot 10^{-5}$: a) $\rho_0 = 0.01$; b) $\rho_0 = 0.05$; c) $\rho_0 = 0.10$.

Equation (9) holds if $\rho \leq \rho^*$, where ρ^* is the root of the equation

$$F(\rho^*) = \alpha^{-1} + F(\rho_0).$$

An analysis shows that this value is attained asymptotically when $\xi \to \infty$. As the following calculations show, the value of ρ^* differs very insignificantly from ρ_*.

In many cases where α is small the term proportional to α can be neglected in (9) (this corresponds to neglect of the pressure decrease along the tube). Then, from (9) we have

$$\xi \approx \frac{1}{\rho_0} - \frac{1}{\rho}, \quad \rho \approx \frac{\rho_0}{1 - \rho_0 \xi}. \quad (11)$$

This expression is a sufficiently good approximation to (9) when $\rho_0 \xi \ll 1 - \rho_0/\rho_*$.

Curves $\xi = \xi(\rho)$, obtained after numerical integration of (9) on a computer, are shown in Fig. 1 for $\alpha = 3 \cdot 10^{-5}$ and $\rho_0 = 0.01$, 0.05, and 0.10.

The dashed lines in Fig. 1 show the relationships $\xi = \xi(\rho)$ derived from (11). In all the calculations here and below we take $\rho_* = 0.60$; the value of ρ^* was always greater than 0.58. It can be seen that a significant difference between the exact solution of (9) and the approximate solution (11) is characteristic only of the region of relatively high ξ and ρ. The investigations show that the values of $\xi(\rho)$ for any ρ increase monotonically with increase in α (very slowly when $\alpha \lessapprox 10^{-4}$–$10^{-5}$ and very rapidly when $\alpha > 10^{-3}$).

Using formula (5), we obtain the following expression for the ratio β of the true hydraulic resistance of the channel to the resistance $(\Delta p)_0$ of this channel with impermeable walls

$$\beta = \frac{\Delta p}{(\Delta p)_0} = \frac{1}{L} \int_0^L \frac{\mu(\rho)}{\mu(\rho_0)} \frac{\rho_0}{\rho} dz =$$

$$= \frac{1}{r} \int_0^r \frac{(\rho_* - \rho_0)^3}{(\rho_* - \rho)^3} \frac{\rho_0}{\rho} d\xi, \quad (12)$$

where

$$(\Delta p)_0 = \frac{8 Q_0 \mu(\rho_0)}{\pi R^4}, \quad r = 2\pi kLR \frac{p_0 - \varphi}{\rho_0 Q_0}. \quad (13)$$

The relationships $\beta = \beta(r)$ corresponding to the curves in Fig. 1 are shown in Fig. 2 (solid curves); the dashed lines give the curves of $\beta(r)$ on the assumption that the approximate expressions (11) are valid. It is clear that with increase in the parameter r the

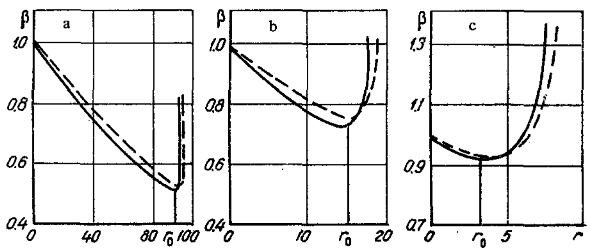

Fig. 2. Relationships $\beta = \beta(r)$ corresponding to curves in Fig. 1.

coefficient of reduction of the hydraulic resistance β decreases at first, reaches a minimum when $r = r_0$, where r_0 depends on ρ_0 and α, and then begins to increase, asymptotically approaching infinity when $\rho \to \rho^*$. The last result is obviously due to the increase in viscosity of the dispersoid with increase in its concentration, which plays the main role at large r. On the other hand, at small r this effect is insignificant and there is a reduction of the hydraulic resistance, as in the case of motion of a uniform viscous liquid through tubes with permeable walls [2]. We note that at low α (of order 10^{-4} or less) the values of β are practically independent of α in the region of ρ of interest.

The pressure loss in the case of a flow of clay solutions in tubes with permeable walls was determined experimentally in [1]. (The most recent experimental data relating to the reduction of hydraulic resistance and the formation of a clay crust on the walls of the channel were given in a paper by R. T. Aliev at the Symposium on the Hydraulics of Drilling and Cementing Fluids, Kiev, April 1967.) The values of r in these experiments corresponded to the region of reduction of hydraulic resistance. Unfortunately, these experiments were not accompanied by careful measurements of the viscosity and a determination of ρ_0. This makes it difficult to test the theory. There is good qualitative agreement, however, between the theoretical and experimental results. For instance, in [1] the dependence of β on the total filtration flow of the liquid phase through the wall and on the pressure drop $p_0 - \varphi$, which are proportional to r, was investigated. These experimental relationships have precisely the same shape as the descending portions of the curves of $\beta(r)$ in Fig. 2.

Experiments also indicate that when sufficiently concentrated disperse systems move through a tube or well in a porous medium a crust consisting of particles of the dispersed phase forms on the walls and this impedes the filtration of the dispersion medium. For a thorough consideration of the dynamics of crust formation we would obviously have to consider the unsteady problem of motion of a complex medium and take into account the interactions of individual solid particles. Here we adopt a phenomenological approach based on the principle of minimum energy dissipation. This approach is often used in the physics of irreversible processes. It was applied earlier [3] to the motion of disperse systems in a study of the wall effect and led to results which agreed with the experimental results.

We assume that the flow produces an "equilibrium" crust corresponding to the completely steady regime and such that the energy dissipation (or pressure drop in the tube) is a minimum. It is clear that the formation of the crust will reduce both the effective tube radius (which becomes $R_* = R - h$, where h is the thickness of the crust) and the proportionality factor k in (3), so that the effective value of k, as can easily be shown, is

$$k_*(h) = \left(\frac{1}{k} + \frac{1}{sh}\right)^{-1}. \qquad (14)$$

Usually $h \ll R$ and only the second effect is significant. In this case Δp reaches its minimum value at the same value of r as the minimum of $\beta(r)$. The relationship between β and k is of the same form as the relationship $\beta(r)$, since $k \sim r$. Using for simplicity the crust thickness averaged over the length of the tube we see that when $r \leq r_0$ no crust at all is formed, but when $r > r_0$, where the value of r is determined from (13) with the value of k from (3), the crust thickness is given by the equation

$$\frac{k_*(h)}{k} = \frac{r_0}{r}. \qquad (15)$$

Thus, the buildup of the crust on the walls leads to a reduction of the value of r corresponding to the region of increase of function $\beta(r)$ to the value r_0 corresponding to the minimum value of β. We note that a more detailed analysis in which the change in crust thickness with distance along the tube axis is taken into account leads to the conclusion that crust formation does not usually begin at the entrance section, but at some cross section further downstream. The mean crust thickness differs slightly from the root of Eq. (15). This gradual buildup of the crust is also typical of oil-well experiments.

We note that the last results apply to systems in which there are no specific attractive forces between the particles, i.e., the spontaneous formation of an internally bound porous structure is impossible. In the latter case, of course, the affinity between the particles would have to be taken into account.

NOTATION

L is the tube length; R is the tube radius; p and φ are the pressure inside tube and external pressure, respectively; Q is the flow of disperse system; q is the density of flow of liquid phase through walls; ρ is the volume concentration of solid phase in system, connected with its porosity ε by the relationship $\rho = 1 - \varepsilon$; k is the proportionality factor in the expression for q in terms of $p - \varphi$, which is proportional to the permeability of the wall material; μ_0 is the viscosity of liquid phase; μ is the effective viscosity of disperse system; h is the thickness of crust on walls; s is the crust permeability per unit thickness. The subscript zero indicates parameters determined at the entrance section of the tube.

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